THE BAROMETER: ITS REDUCTION TO SEA LEVEL.

By A. G. Howard.

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The subject of my paper is the reduction of an indicated atmospheric pressure at an elevated station to that which it would be were the earth below that station removed, and the observation of pressure made at the sea level. The object of thus reducing the indicated pressures to one datum, is for the purpose of charting a number of simultaneous observations for comparison and for the drawing of isobars or lines of equal pressure. The datum employed is usually the mean sea level.

The true formula for this reduction has not yet been developed, and it is my firm opinion, that as far as reductions of pressures on isolated mountains are concerned no formula will ever be produced for giving the true sea-level equivalent. The local eddies, and alterations of temperature, pressure and humidity are such, in such a position, not only at the station itself but at varying heights, that the formula which would be correct at one time would give very erroneous results at another.

But on elevated table lands the conditions are totally different; in such a position there are no local eddies whirling up or down, no atmosphere of varying temperature and humidity below; whatever the reduction is calculated to be, it can only vary with the pressure; the temperature correction has already been applied to the column of air above the station; consequently the conditions are very much simplified.

The formula of La Place takes into consideration the mean temperature of the column of air below the station, but this is not always so easy to find. The actual mean is not always the mean of the two stations. The temperature variable makes a great difference in the reduced reading, and is often very misleading; so much so was this the case, that in the year 1872 the Chief Signal Officer in America ceased to use this variable for high stations and substituted an annual constant. But still at stations of from 2,000 to 5,000 feet in altitude, the same trouble was experienced by using the temperature variable, and in the year 1881 a system for all stations was introduced, by using monthly constants of reduction to sea level based on monthly normal pressures and temperatures, but this has features quite as objectionable as any other mode.

At the third meeting of the International Meteorological Committee held at Paris in 1885, the late General Hazen of the American Signal Service submitted a scheme for the reduction to sea level, by using for the temperature variable, the mean of the observed temperature and those of the two preceding observations where tri-daily observations are taken, and where only one observation a day is telegraphed to send the mean of the hours 7, 2, 2 and 9. The idea as far as I can see is to use the progressive mean of the day. Mr. Leyst* pointed out the error of this mode of reduction, which instead of obliterating the diurnal curve increased and misplaced it. He still considers that the formula of La Place should be adhered to, and the constants and variables revised according to the light of modern science, and furthermore that the formula should be critically examined with a view to constructing tables to avoid the use of logarithms. He professes to have solved this problem himself. This may be; but I do not believe, even at the best, that it will be reliable for great heights.

Rühlmann, who has corrected the formula of La Place, makes the temperature answerable for all errors of reduction and alters his constants accordingly. But this, I think, is not a very scientific proceeding.

Leyst again points out that humidity has a considerable influence on the pressure. So, I say, has the direction of currents, whether they are rising or falling. The reductions in the centres of cyclones and anti-cyclones would be totally different.

Latitude also makes a difference. According to Leyst; at an altitude of 6,500 feet, a difference of 80° in latitude makes a difference of ·034 of an inch in the pressure. At the same altitude and with a temperature of 20° centigrade, a difference in the relative humidity of eighty-seven, causes a difference in pressure of ·063 of an inch. Hence these factors are to be taken into account, and when we consider that temperature and humidity vary considerably at different heights, and that at a high station such variations are totally

* Pawlowsk Observatory.

unnoticed, I fail to see that the formula of Leyst will be of greater value than that of La Place. As I said before, the conditions are totally different on elevated table lands, and it is principally to these I shall devote my paper.

No formula will ever be reliable for mountain stations higher than 2,000 feet, and for these La Place's as corrected is, and always will be, the best.

In reducing to a datum, it is not absolutely necessary to use the sea level, almost any horizontal plane can be used, and the readings reduced either up or down to it.

But there is a great difference in the results to be obtained by the two modes of reduction. By reducing up we obtain a chart cutting through various conditions of atmosphere, owing to the datum plane being at varying heights above the ground; at some points it will touch the ground, and at others, be at varying distances above, 500 feet, 600 feet, or 5,000 feet or more.

Consider the varying conditions of the atmosphere at these different heights.

I mentioned in a former paper that the storm-bearing stratum of the atmosphere flows up and over the land where the slope is gradual, but flows round mountain peaks; so that over the Colony, the height of this stratum would be approximately the same at Cape Town and at Kimberley. Now consider this stratum to be 3,000 feet thick, and let us assume it is very dense, while above flows a current of warm air. A chart plotted on the plane of Kimberley would not represent the actual condition of the atmosphere if reduced up from surface temperatures.

Upon a chart reduced to sea level, the pressure at Kimberley is least, consequently the gradient is from Cape Town and the wind is south-west.

But by reducing up to the plane of Kimberley 4,042 feet above sea level, using the surface temperature, minus the diminution for 2,021 feet, the reduced pressure would be much too low, the gradient being from Kimberley, so that the wind would be shewn to flow to a high area instead of to a low. This will be better understood if we consider the temperature at Cape Town and Kimberley to be 60°. The correction reduced down for Kimberley would be with a variable of 67°, but for Cape Town reduced up it would be 53° or a difference of 14°, causing a difference in pressure of ·090 of an inch. Thus if by reducing down Cape Town reads 30·05 inches and Kimberley 30·00 inches, by reducing up Cape Town will read 25·94 inches, and Kimberley 25·98 inches, thus reversing the gradient.

In reducing down to the sea level the conditions are totally different. It is true that this is an attempt to delineate an impossibility, but of the two impossibilities it is the least objectionable.

If we assume the whole of the solid earth above the mean sea level to be so much atmosphere, perfectly tranquil and unaffected by temperature, currents or any movement, and in layers of diminishing density, the atmosphere itself rolling over it as it does over the solid earth. If we again assume that the weight of the atmosphere above this mass affects it by compression, so that the surface pressure shall bear a proportional ratio to that at the sea level, we have data for constructing reduction tables, for varying heights, provided we know the decreasing ratio of atmospheric density.

It has been demonstrated that the density of the atmosphere decreases with the squares of the distances, and that the number representing this decrease of density obtained by dividing the pressure at any altitude into 30 inches, increases in the proportion of the power of the quotient of altitudes, or in other words, if B represents the upper pressure then log. $\frac{30}{B}$ increases directly as the altitude, so that by applying a correction to the height, the log. of $\frac{30}{B}$ will be obtained.

Hence we have the resolvable equation—

$$H = \log. \frac{30}{B} kg \qquad (1)$$

Where H is the altitude in feet, and B the pressure at that altitude, k is a constant and g the latitude correction.

To find k, let a represent log. $\frac{30}{B}$. Then, omitting g we have, $ak = H$, and consequently—

$$k = \frac{H}{a} \qquad (2)$$

So that by taking the mean of a number of readings at two stations, the one at sea level and the other at a known altitude, we can easily work out k. I have calculated this from readings at Graham's Town and Port Alfred, and find it to be 65,000. This number varies slightly with the sea-level pressure, varying from 64,900 to 65,100, from a pressure of 29 to 30 inches. It is also the same for all altitudes.

The latitude correction has now to be considered. The weight of a body on the earth, decreases from the poles towards the equator in the proportion of $W\left(1 - \frac{\cos^2\phi}{289}\right)$ so that the weight of the atmosphere below any altitude, or the term $30 - B$ will *increase* in that proportion towards the poles, hence the latitude must be replaced by the co-latitude, and as the cosine of the co-latitude is the same as the sine of the latitude, we have the latitude correction as $1 + {\cdot}00346 \sin^2\phi$. Colonel Clarke from pendulum observations has altered the co-efficient to ·002606.

The formula for heights based on the foregoing is

$$H = 65{,}000\ (1 - {\cdot}002606 \sin^2\phi) \log. \frac{B}{B'} \qquad (3)$$

and for reduction

$$\log. \frac{B}{B'} = \frac{H\ (1 + {\cdot}002606 \sin^2\phi)}{65{,}000}. \qquad (4)$$

The reduction to the gravity of latitude 45° is given by La Place as $1 + {\cdot}00284 \cos 2\theta$.* Professor Archibald, from Colonel Clarke's observations, has altered this to $1 - {\cdot}002606 \cos 2\phi$.

Between Cape Town and Kimberley this correction for gravity makes a difference of ·013 of an inch, so that the reduced readings at the latter should have this correction applied subtractively.

The foregoing gives the argument upon which I have considered this question. Of course the formula given will only be applicable for stations on elevated plateaus. For mountain stations they will not answer, as temperature and humidity have such different effects at such stations to what they have on elevated table lands. Moreover the atmosphere between the station and the foot of the mountain has to be taken into consideration with all its varying conditions. For these stations I would introduce the temperature correction $t' = t + \frac{h}{2c}$ where t represents the mean temperature of the date of observation for a great number of years, and $\frac{h}{2c}$ the variation due to altitude, c being the number of feet in which the temperature decreases one degree. This variable c is different for different latitudes and temperatures.

In the tropics the perpetual snow limit is 16,000 feet above the sea level. Taking this as a datum we find that with a sea level temperature of 80°, the decrease is 1° in every 333 feet of altitude, but with a

* Co-Latitude.

sea-level temperature of 60° the decrease is 1° in every 571 feet. The height of the limit of perpetual snow decreases towards the poles in the proportion of the squares of the cosines, and the altitude wherein the temperature decreases 1° will vary accordingly.

By the term tropic I mean that parallel of latitude where the sun shines vertical. The calculation for decrease must be made from this parallel and not from the equator, bearing in mind that solar action lags behind nearly a month. The decrease of temperature due to altitude which I have mentioned is the mean decrease ; the nearer the surface of the earth, the quicker does the change take place. This question of decrease of temperature ought to be gone into practically, and I should like to see it done here. We have Table Mountain and Lion's Head. Why not utilize them ? A wet and dry bulb on Signal Hill would help. Where there's a will there's a way, and I should like to see South Africa take off the palm in this question.

I will now pass on to some practical results. My first attempt to construct charts from barometer readings at high altitudes, was early last year. I compiled thirteen charts for 1885, twelve monthly means and a yearly one. I used Hazen's tables of reduction, and was very successful.

On attempting to plot daily charts, I found that temperature, which had been reduced to a mean in the monthly ones, seriously affected the reductions. One station had too high a reading, another too low, consequent upon the height of the thermometer, and the greater the altitude the greater the error.

I was so annoyed at this, that I destroyed the whole of them, and plotted new ones, using only those reductions for a temperature of 62°. By these means the differences were done away with, and I came to the conclusion that temperature had nothing to do with a formula for reduction on elevated plains. I then began to work out tables of my own.

The result of my work I now place before you. I do not assert it is perfection, nor do I say the formula is new. A new formula is not wanted. As I said before, mine is to be used solely in reducing from table lands, and by using it we do away with the discrepancies due to the temperature variable, and with the anomaly of a temperature correction where no atmosphere exists.

I have compiled tables of my own for all the principal places in the Colony from which returns are received, and have been successful so far.*

* Charts for the first six days of January and of July, 1885, were exhibited at the meeting.

In conclusion I will give two cases, to show how erroneous it is to take temperature into consideration on elevated table lands. Assume two stations ten miles apart, situated on a plain, both having the same altitude and barometer reading, while the temperature of one is 10° warmer than the other. Upon a chart plotted on the plane of these stations both pressures would be alike, which is the truth, but upon their being reduced to sea level they are totally different, which is an untruth, but by my mode of reduction the truth would exist, even at the sea level.

The other case is this. Often when pressure has been very steady all over the Colony, the diurnal range of temperature at an elevated station has been very great. Now if the readings had to be reduced to a steady sea level pressure the upper barometer must have oscillated very much, but as such was not the case, the effect of temperature must have been *nil*. Major Pinto, a traveller who crossed Africa from Benguela to Natal, notices this. He says : "Whoever bestows any attention upon the meteorological observations I publish, will see that the atmospherical changes in this part of Africa have but a slight, if any, influence upon the pressure, which remains the same amid the most sudden variations."

And again he says : "An investigation of these tables will shew the great uniformity of the barometrical oscillations and the enormous inequalities of temperature and humidity of the air in the countries to which they refer." The country Major Pinto refers to is the great table land of Central Africa, far from all influences of atmosphere *below* the places of observation.

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